CONTROL OF COOLING DURING HARDENING TAKING INTO ACCOUNT THE EFFECT OF STRESSES ON PHASE TRANSFORMATIONS

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A correct mathematical formulation is proposed for the problem of controlling cooling during hardening of elastoplastic samples. The optimal controls for surface and through hardening were found from a numerical experiment.

1. It is well known that quench hardening of structural carbon steels depends strongly on the rate of cooling. The distribution of cooling rates in the sections of quite brittle real machine parts is extremely nonuniform. As a result, structure formation proceeds differently at different points in the sample. On the other hand, when the rate of surface cooling is increased the stress gradient increases. Obviously, the nonuniformity of structure formation over the cross section of the sample affects the stress field, but even the stresses can have a definite effect on structure formation in the process of hardening [1, 2]. It is desirable to take this effect into account in order to obtain more complete information about the character of structure formation during cooling. In this connection there arises the problem of determining the optimal rate of surface cooling which gives the desired structure of the surface layer.

In this paper, with the help of a mathematical experiment on a computer, we solve this problem of controlling the technological process. Problems of this class are improperly posed, and in constructing a stable algorithm for finding the control function we make use of the theory of regularization [3]. This makes it possible to obtain a more detailed solution than in the case of [4], where this problem is also studied. On the other hand, in order to calculate variational problems of temperature and mechanical fields as well as the distributions of the microstructural components of the material we employ a quite complete physicomathematical model constructed on the basis of [1, 5, 6]. This is a self-consistent "evolutionary system" of equations which includes the following: a) nonlinear heat-conduction equation with convective transfer and radiative boundary conditions and taking into account the release of latent heat accompanying phase transitions; b) nonlinear equations of the theory of plastic flow for calculating stress and strain fields, including the structural component of the strain tensor and a component due to the plasticity of perlite and martensite transformations; c) working formulas for determining the perlite, martensite, and austenitic components based on the Jones-Mell hypothesis with a correction for the nonisothermal nature of the process of decomposition of austenite and taking into account the effect of stresses on this process; and d) at the stage of induction heating preceding hardening the system also includes Maxwell's equations in the quasistationary approximation.

We employed iterative difference methods, similar to those of [4, 7], in order to solve this problem. We now consider the mathematical formulation of the control problem.

2. Let $\xi_p(r, t)$ be the distribution of the fractional component of the perlite phase accompanying decomposition of austenite into perlite and martensite. Then the condition of maximum content of the martensite component in the layer $[R^*, R]$, $0 \leqslant R^* < R$ (R is the radius of the sample) corresponds to a minimum of the integral

$$\Phi_1 = \int_{R^*}^{R} \xi_p(r, t)\, dr.$$

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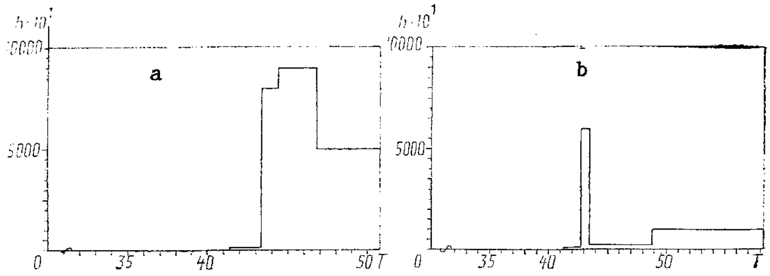

Fig. 1. Step functions controlling the surface cooling for regimes A (a) and B (b). h, W/(m²·C); T, sec.

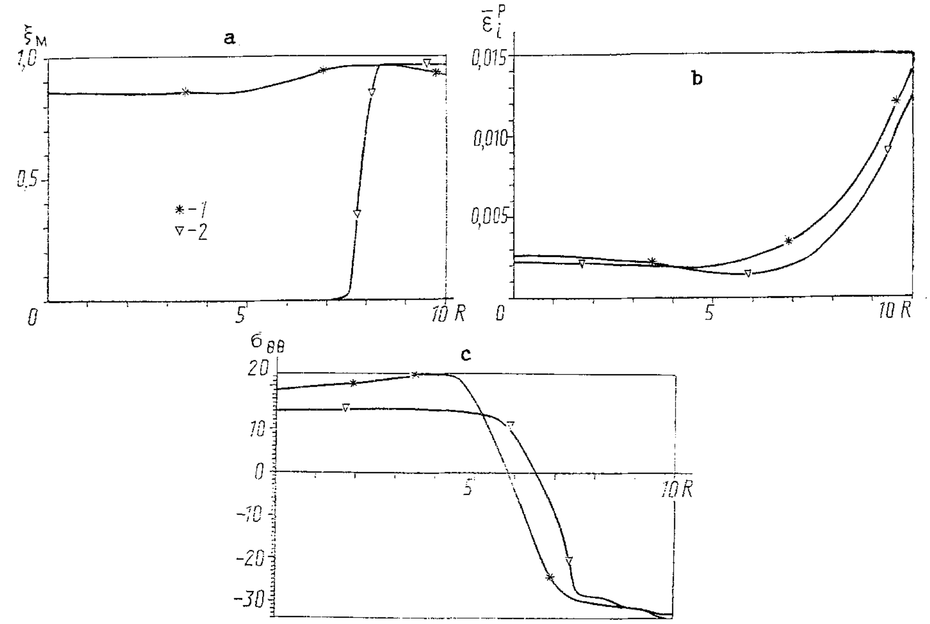

Fig. 2. Distribution of the martensite component ξ_m (a); accumulated plastic strain $\bar{\varepsilon}_j^P$ (b); residual tangential stresses $\sigma_{\theta\theta}$, kg/mm² (c): 1) regime A, 2) regime B. R, mm.

This condition must be supplemented with conditions following from the interaction of structure—formation and elastoplastic-deformation processes. The austenite—martensite transformation in the surface layer results in a positive change of volume Δv, which in turn causes compression at interior points of the sample and results in inversion of the axial and tangential stresses in the surface layer and compressive stresses at interior points, a phenomenon noted in practice and obtained as a result of mathematical modeling [2, 8]. In this connection there arise a number of problems.

On the one hand, in the case of plastic strains this phenomenon can result in the fact that the tangential tensile stresses in the surface layer will be of a residual character. These stresses can lead to the formation and development of cold cracks on the surface of the sample. In order to avoid such effects it is desirable to impose with the help of the functional

$$\Phi_2 = \int_0^R \bar{\varepsilon}_i^P(r,\ t)\, dr$$

the condition that the accumulated plastic strain $\bar{\varepsilon}_i^P$ be minimum at all points of the sample.

We note that the last condition does not always solve the problem of crack formation, and after the variational problem including this condition is solved the quality of the obtained sample must be checked. This can be done with the help of the threshold value of the fracture toughness K_{Irth}, determined from the experiment of [9], as well as the admissible value of the crack depth ℓ according to the state standard and obtained by solving the problem of the residual distribution $\sigma_{\theta\theta}$ of tangential stresses in the surface layer. Then the quality of the sample can be checked according to the condition [9]

$$K_1 = 1{,}12\sigma_{\theta\theta}(R)\sqrt{\pi l}/\Phi \leqslant K_{\mathrm{Irth}},$$

where Φ is a function that takes into account the shape of the crack (for surface defects in blanks $\Phi = 1$).

On the other hand, the most characteristic aspect of the structure formation process is that the perlite fraction increases and the martensite fraction decreases away from the surface toward the center of the sample [1, 10]. But the compressive stresses at interior points of the sample can result in an anomalous increase in the martensite and decrease in the perlite components, a phenomenon observed in practice and called the "inverse hardening effect." The reasons for the appearance of this effect and measures taken to prevent it were discussed in [8, 10]. Here we can impose two different conditions, depending on the value of the parameter R^*. In the case of through martensitic hardening ($R^* = 0$) it is desirable to impose with the help of the following functional Φ_3 (regime A) the condition that the radial distribution of the perlite component be monotonic and the condition that the perlite component can increase at interior points of the sample:

$$\Phi_3 = \int_0^R \varphi(r, t)\, dr,$$

$$\varphi(r, t) = \begin{cases} \left(\dfrac{\partial \xi_p}{\partial r}\right)^2, & \text{if} \quad \dfrac{\partial \xi_p}{\partial r} < 0, \\ 0, & \text{if} \quad \dfrac{\partial \xi_p}{\partial r} \geqslant 0. \end{cases}$$

In those cases when formation of 90% martensite is required only in the surface layer $\delta = R - R^*$ (δ = 10-15% R), it is useful to impose a stricter condition with the help of the functional Φ_3^* (regime B) on the formation of the perlite component:

$$\Phi_3^* = \int_0^{R^*} [1 - \xi_p(r, t)]\, dr.$$

The quantities Φ_i (i = 1, 2, 3) and K_1, introduced above, are functionals of the parameter controlling the process. For this parameter we take a time-dependent heat-transfer coefficient $h(t)$ under the condition of heat transfer at the surface taking into account convection and radiation:

$$-\lambda(T)\frac{\partial T}{\partial r}\bigg|_{r=R} = h(t)(T - T_0)\bigg|_{r=R} + \hat{\sigma}\hat{\chi}(T^4 - T_0^4)\bigg|_{r=R},$$

where $\lambda(T)$ is the thermal conductivity, $\hat{\sigma}$ is the Stefan–Boltzmann constant, $\hat{\chi}$ is the emissitivity factor of the body, and T_0 is the temperature of the surrounding medium.

In accordance with the concept of regularization we employ an approximation of the function $h(t)$, belonging to some compact set. Let $h(t)$ be defined in some segment $[t_0, \hat{t}]$, where t_0 is the start of the process and $\hat{t}$ is an a priori fixed quite large quantity — the moment at which the sample has completely cooled. We divide the time segment into N equal parts $[t_{j-1}, t_j]$, $j = \overline{1, N}$, and define on this set a piecewise-constant bounded function:

$$h(t) = h_j,\ t \subset [t_{j-1},\ t_j],\ \bar{h}_j \leqslant h_j \leqslant \bar{\bar{h}}_j,\ j = \overline{1, N}.$$

We assume that the values of $\bar{h}_j$ and $\bar{\bar{h}}_j$ are known a priori from the experimental data set. Thus the control function h(t) sought on each time segment $[t_{j-1}, t_j]$ is determined on some compact set $P_j = \{h_j \mid \bar{h}_j \leqslant h_j \leqslant \bar{\bar{h}}_j\}$. Union of such sets $P = \bigcup_{j=1}^{N} P_j$ is a compact set, and therefore on the segment $[t_0, \hat{t}]$ h(t) belongs to the compact set $h(t) \subset P$.

Now the optimal equation can be sought as the solution of the variational problem

$$\inf_{h \supset P} \Phi[h] = \inf_{h \supset P} \{\gamma_1 \Phi_1[h] + \gamma_2 \Phi_2[h] + \gamma_3 \Phi_3[h]\}$$

under the condition $K_1[h] \leqslant K_{1\text{rth}},\ t = \hat{t}$. Here γ_i are weighting factors, introduced taking into account the difference in the dimension of the functionals ($\gamma_i = 1/|\Phi_i|,\ i = \overline{1,3}$). In the case $R^* = R - \delta$ the last term is replaced with $\gamma_3^* \Phi_3^*[h]$.

We now examine the elements of the algorithm for solving the problem posed.

3. In practice, it is found that it is more efficient to minimize the functional Φ successively on the time segments $[t_{j-1}, t_j]$:

$$\inf_{h_j \subset P_j} \Phi[h] = \inf_{h_j \subset P_j} \left\{ \sum_{i=1}^{3} \gamma_i \Phi_i[h] \right\}$$

under the condition $K_1[h] \leqslant K_{1\text{rth}},\ t = \hat{t}$.

Qualitative analysis of the rate-of-cooling fields, performed based on the scheme of optimal cooling in [2], leads to the conclusion that in order to avoid "sliding down" into a local minimum at the very beginning of the cooling process (when, for example, $\Phi_1 = 0$, $\Phi_2 = 0$, $\Phi_3 = 0$) the maximum admissible rate of cooling for which plastic strain does not yet occur should not yet occur should be provided. For this, it is sufficient to replace the functional Φ_2 with the functional Φ^*_2:

$$\Phi_2^* = \begin{cases} \Phi_2, & \text{if } \Phi_2 > 0, \\ \Phi^*, & \text{if } \Phi_2 = 0, \end{cases}$$

where Φ^* is a constant, taken as the largest value $\max_{h_j \subset P_j} \Phi_2$. Since we are studying only deformations for which $\bar{\varepsilon}_i^P \ll 1$ it is sufficient to set $\Phi^* = R$.

Thus the initial problem reduces to the problem of minimizing the functional Φ with respect to one variable — finding the element h_j successively on each segment $[t_{j-1}, t_j]$. Since $h \subset P_j$, the sequence $\{h_j^{(n)}\}$ minimizing the functional Φ turns out to be regularized [3]. Since Φ is nonconvex on the entire set P, the minimizing sequence is constructed by a complex method: at the first stage the method of covering of the set P_j by a uniform grid is employed; at the second stage the method of steepest descent is employed [11]. The results presented below confirm that our minimization algorithm is efficient.

4. Calculations in the case of the two optimization regimes (A) and (B) were performed for a 20 mm in diameter No. 40 steel sample, heated uniformly up to the temperature of complete austenization 860°C. Figure 1 shows the control functions which are the solution of such problems.

In accordance with these results (for simple carbon steels) at the initial stage of cooling, before the austenitic-perlite transformation starts, forced cooling is not recommended (this will prevent the development of significant plastic deformations). At the next stage at which the surface layers pass through the temperature of austenitic-perlite transformation the rate of cooling should be significantly increased in order to avoid the formation of a perlite structure in the surface layer. In the case of through martensitic hardening (see Fig. 1a) the time interval during which the cooling rate is high should be quite long and the rate of cooling should decrease somewhat after all points of the sample have passed through the temperature of the austenite-perlite transformation. In the case of surface martensitic hardening (Fig. 1b) there should be a quite sharp "peak" in the rate of cooling. The "trough" following this peak prevents reverse hardening, associated with the back effect of the stress field on the formation of perlite structure. After the austenite-perlite transformation is competed, as in the first case, the rate of cooling must be increased.

The distribution of the martensite component in the hardened sample (Fig. 2a) indicates the effectiveness of both hardening regimes. However the development of plastic deformation in this sample could not be avoided (see Fig. 2b). It should be noted that plastic strains appear at the moment of forced cooling of the sample — at the stage when the surface layers pass through the process of decomposition of austenite. The quality of the sample for both hardening regimes was checked with the help of the distribution of residual tangential stresses (Fig. 2c). Since compressive stresses ($\sigma_{\theta\theta}(R) < 0$) form on the surface of the sample while tensile stresses form in the interior zones (at a depth up to one-half the radius), there is no possibility for cold cracks to form; this confirms that our solution of the optimization problem is efficient.

Thus the methods developed for solving the optimization problem can be used to solve a wide class of problems concerning the optimal control of the hardening process. The problems involve choosing the depth δ of the hardened layer and regulating the level of residual stresses or the magnitude of the accumulated plastic strain.

The cooling regimes obtained could aid in conducting more efficient and optimal hardening processes under production conditions for a wide class of steels.

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